Supporting Information SI2

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SI Text 2

Skew product system for the E(3)-extension. In the case of $\mathbf{E}(3) = \mathbf{SO}(3) \ltimes \mathbb{R}^3$, it is convenient to make the identification $\mathbf{SO}(3) \cong \mathbf{SU}(2)/\{\pm I_2\}$ where $\mathbf{SU}(2)$ is the special unitary group of 2×2 complex matrices with determinant 1. Such matrices have the form

$$A = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix},$$

where $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$. We also identify $\mathbb{R}^3$ with 2×2 complex matrices

$$v = \begin{pmatrix} ia & b \\ -\bar{b} & -ia \end{pmatrix},$$

where $a \in \mathbb{R}$, $b \in \mathbb{C}$. The action of $\mathbf{SO}(3)$ on $\mathbb{R}^3$ is given by $A \cdot v = AvA^{-1}$. The skew product takes the form

$$(x, A, p) \mapsto (f(x), Ah(x), p + A \cdot v(x)), \quad (1)$$

where $A, h(x) \in \mathbf{SU}(2)$ and $p, v \in \mathbb{R}^3$. Hence

$$p(n) = \sum_{j=0}^{n-1} A_j \cdot v(x_j), \quad A_j = h(x_0)h(x_1)\cdots h(x_{j-1}). \quad (2)$$

For the numerics, we choose

$$a(x) = 2 + x \qquad \text{and} \qquad b(x) = (1 + i)(2 + x),$$

and we represent

$$h = \begin{pmatrix} \cos(c_0)\exp(ic_1) & \sin(c_0)\exp(ic_2) \\ -\sin(c_0)\exp(-ic_2) & \cos(c_0)\exp(-ic_1) \end{pmatrix},$$

where the functions c_i are chosen to be piecewise constant on the subintervals $[0, \frac{1}{2}]$, $[\frac{1}{2}, \frac{3}{4}]$, $[\frac{3}{4}, 1]$ with values chosen from a uniform distribution in the interval $(\pi/5, 4\pi/5)$ (so 9 different values are chosen at random). In Figure 4 we show plots of the translation variables in the (p_1, p_2)-plane and the process $p_1(n)$ as a function of time for strongly and weakly underlying dynamics. The anomalous Lévy type behaviour is clearly visible in the weakly chaotic case.

The anomalous diffusion linked to underlying weakly chaotic dynamics of the Pomeau-Manneville map can be motivated by inspecting the skew product (1) as follows: Let us decompose $v(0) = v^{\parallel} + v^{\perp}$ where $v^{\parallel}$ denotes the component along the axis of rotation of $h(0)$ and $v^{\perp}$ denotes the component perpendicular to the axis of rotation. (In the notation of **[8]**, 0 corresponds to x_0 and $v^{\parallel}$ corresponds to $v_1(x_0)$.) Then, within the laminar phase with x_j close to the indifferent fixed point for N_0 iterates say, the translation variables p are augmented by approximately $v^{\parallel} N_0$ (cf. Figure 1). Noting the expression for A_j in equation (2), in the weakly chaotic case a requirement for the occurrence of anomalous diffusion is that $v(0)$ has a nonvanishing component $v^{\parallel}$ along the axis of rotation of $h(0)$.

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